Asymptotic Behavior for the Extreme Values of a Linear Regression Model

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Abstract

We consider a class of linear regression model Y_t defined by $Y_t = aX_t + b + \zeta_t$ where (ζ_t) is a white noise process. We assume that (X_t) and (ζ_t) are independent and the distribution function of ζ_t is known. We are interested by the behavior of $\max_{1\leq k\leq n} Y_k$. We show that the extreme value theory for the process (Y_t) is the same that the one of the extreme value theory for the process (X_t) under specific conditions.

keywords and phrases: Extreme value theory, Poisson random measure, Point process, Regression model.

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1 Introduction

In this paper we are interested to study the asymptotic behavior of the maxima for a regression model defined by the following scheme:

$$Y_t \;=\; aX_t \;+\; b \;+\; \zeta_t \tag{1}$$

where (ζ_t) is a white noise process and (X_t) a sequence of independent and identically distributed random variables (iid rvs) independent of (ζ_t).
Here we assume that the distribution of ζ_1 is known. We assume too that $X_1,\ X_2,\ \ldots$ are iid rvs with common distribution function F belonging to the extreme value domain, see Gnedenko (1943).
Related work on extremes of such model is Horowitz(1980) who considered the following model for daily ozone maxima (Y_t):

$$lnY_t \;=\; f(t) \;+\; \zeta_t \tag{2}$$

where $f(t)$ is a deterministic part and ζ_t is a gaussian stationary autoregressive process. The limit theory for processes of the form

$$Y_t \;=\; f(t) \;+\; h(t)\zeta_t \tag{3}$$

where $h(.)$ is positive and periodic and (ζ_t) is a stationary process satisfying certain mixing conditions has been studied by Ballerini and Mc Cormick (1989). Niu (1996) studied the limit theory for extreme values of a class of nonstationary time series with the form

$$Y_t \;=\; \mu_t \;+\; \zeta_t, \quad \zeta_t \;=\; \sum_{j=0}^{\infty} c_j\, Z_{t-j} \tag{4}$$

where $Z_t \;=\; \sigma_t\eta_t$ and $(\eta_t,\ t \in \mathbb{R})$ is a sequence of iid random variables with regularly varying tail probabilities. See also Resnick (1987) and Kallenberg (1983) for point process results relevant to the present setting.

Before going further let us recall the following result which is the basis of classical extreme value theory.

Theorem 1 *(Fisher-Tippett theorem, limit laws for maxima)*
Let (X_n) be a sequence of iid rvs. If there exist two sequences $(a_n > 0)$, (b_n) and some non-degenerate df H such that

$$a_n^{-1}\,(\max_{1\leq k\leq n} X_k \;-\; b_n) \;\xrightarrow{d}\; H, \tag{5}$$

then H belongs to the type of one of the following three dfs :

$$\begin{array}{llcll} \textit{Gumbel} & \Lambda(x) & = & exp(-e^{-x}), & x \in \mathbb{R}, \\ \textit{Fréchet} & \phi_\alpha(x) & = & exp(-x^{-\alpha}), & x > 0,\ \alpha > 0, \\ \textit{Weibull} & \psi_\alpha(x) & = & exp(-(-x)^{\alpha}), & x \leq 0,\ \alpha > 0. \end{array}$$

■

From now on we refer to the centering constants a_n and the normalising constants b_n jointly as norming constants.

We say that the rv X (the df F of X) belongs to the maximum domain of attraction of the extreme value distribution H if there exist constants $a_n > 0$ and b_n such that (5) holds. We write $X \in D(H)$ $(F \in D(H))$.

Our main result is the theorem 2 when F belongs to the Fréchet domain and the theorem 3 when F belongs to the Gumbel domain. We begin to establish some technical lemma. Applications are provided in a forthcoming paper.

2 Main result

In this section we assume that the distribution function F belongs to the Fréchet domain or the Gumbel domain.

We show that the asymptotic distribution of the extreme values of the variable Y is the same as the one of the variable X if $a > 0$ in (1).

2.1 Preliminaries

We introduce a technical proposition which permit us to deal with a model simpler than (1) using a linear transformation

Proposition 1 *Let* (V_i) *be a sequence of iid rvs,* $(c_n > 0)$ *and* (d_n) *two sequences of* $\mathbb{R}$ *such that for all continuity point* x *of* H*,*

$$P[\max_{1 \le k \le n} V_k \le c_n x + d_n] \longrightarrow H(x),$$

where H *is a non degenerated df. If* f *is an increasing linear function and if* $V_i' = f(V_i)$ *then*

$$P[\max_{1 \le k \le n} V_k' \le c_n' x + d_n'] \longrightarrow H(x),$$

with

$$c_n' = \frac{c_n}{f'(d_n)} \quad \text{and} \quad d_n' = f(d_n).$$

■

Let us now consider the linear transformation :

$$X_t' = aX_t + b$$

where $a > 0$. The proposition shows that $\max_{1 \le k \le n} X_k$ and $\max_{1 \le k \le n} X_k'$ have the same asymptotic behavior. If X belongs to $D(H)$ with the norming constants a_n et b_n then using the previous result, we get

$$P[\max_{1 \le k \le n} X_k' \le a_n' x + b_n'] \longrightarrow H(x),$$

with $a'_n = aa_n$ and $b'_n = ab_n + b$: (thus it suffices to put $a = 1$ and $b = 0$ in (1).

¿From now on, we consider the process (Y_t) defined by the following scheme

$$Y_t = X_t + \zeta_t \tag{6}$$

where (X_t) is a sequence of iid rvs with distribution function F and (ζ_t) is a white noise process independent of (X_t).

2.2 Fréchet domain

Assume now that the distribution function F which characterizes the r.v. X of the model (6) belongs to the Fréchet domain, i.e., $F \in D(\phi_\alpha)$ with $\alpha > 0$. Denote

$$a_n = F^{-1}(1 - \frac{1}{n}) = n^{\frac{1}{\alpha}} L(n), \quad \text{and} \quad b_n = 0$$

where L is a slowly varying function at ∞, i.e. $\lim_{x\to+\infty} \frac{L(tx)}{L(x)} = 1$, $t > 0$, F^{-1} the generalized inverse function of F defined by

$$F^{-1}(y) = \inf\{x \in \mathbb{R},\ F(x) \geq y\}.$$

We also recall the following tail balancing condition for a stationary process (ζ_t) given in Davis and Resnick (1985)

$$\lim_{x\to+\infty} \frac{P\{\zeta_1 > x\}}{P\{|\zeta_1| > x\}} = \pi_0, \qquad \lim_{x\to+\infty} \frac{P\{\zeta_1 < -x\}}{P\{|\zeta_1| > x\}} = 1 - \pi_0, \tag{7}$$

where $0 < \pi_0 \leq 1$.

Let $\mathcal{E}$ be the Borel σ-field of subsets of a set $E \subset \mathbb{R}^k$. For $x \in E$ and $A \in \mathcal{E}$, we define the measure ϵ_x on $\mathcal{E}$ by

$$\epsilon_x(A) = \begin{cases} 1, & \text{x} \in A \\ 0, & \text{x} \notin A. \end{cases}$$

Let $\{x_i,\ i \geq 1\}$ be a countable collection (not necessary distinct) of points of the space E. A point measure m is defined to be a measure of the form $m = \sum_{k=1}^{\infty} \epsilon_{x_i}$ which is nonnegative integer valued and finite on relatively compact subsets of E. The class of point measures is denoted by $\mathcal{M}_p(E)$. Let also μ be a Radon measure on $\mathcal{E}$, a Poisson random measure with mean measure μ will be denoted by PRM(μ).
Now we introduce :

$$N_n = \sum_{k=1}^{\infty} \epsilon_{(\frac{k}{n},\ a_n^{-1} Y_k)},$$

and

$$N_n^{(1)} = \sum_{k=1}^{\infty} \epsilon_{(\frac{k}{n},\ a_n^{-1} X_k)}, \qquad N_n^{(2)} = \sum_{k=1}^{\infty} \epsilon_{(\frac{k}{n},\ a_n^{-1} \zeta_k)}.$$

Thus, N_n, $N_n^{(1)}$ and $N_n^{(2)}$ are PRM. The two next lemma give the convergence of these processes. Part 1) of lemma 1 is due to Resnick(1987).

Lemma 1 *1.) Let (X_t) be a sequence of iid rvs with common distribution F belonging to $D(\phi_\alpha)$ with $\alpha > 0$. Suppose $F(0) = 0$ so that $X_i > 0$ a.s. Then*

$$N_n^{(1)} \xrightarrow{d} N_1 \qquad as\, n \longrightarrow +\infty,$$

in $\mathcal{M}_p([0,\ \infty) \times (0,\ \infty])$, where N_1 is a PRM($\lambda \times \nu_1$) with λ the Lebesgue measure on $[0,\ \infty)$ and $\nu_1(x,\ \infty] = x^{-\alpha}, x > 0$.

2.) Suppose that the sequence of iid rvs (ζ_t) satisfies the tail balancing condition specified in (7) and the following condition for $x > 0$

$$n\bar{F}_{|\zeta|}(a_n x) \longrightarrow 0, \qquad as\, n \longrightarrow +\infty. \tag{8}$$

Then

$$N_n^{(2)} \xrightarrow{d} N_2 \qquad as\, n \longrightarrow +\infty,$$

in $\mathcal{M}_p([0,\ \infty) \times]-\infty,\ +\infty[\backslash\{0\})$, where N_2 is a PRM($\lambda \times \nu_2$) with $\nu_2 \equiv 0$. ∎

Proof : The proof of 1) is identical to that of corollary 4.19 of Resnick (1987). The proof of 2) follows from proposition 3.21 of Resnick (1987) by showing that

$$nP\{a_n^{-1}\zeta_1 \in .\} \xrightarrow{\nu} 0 \quad \text{as } n \longrightarrow +\infty, \tag{9}$$

where $\xrightarrow{\nu}$ denotes vague convergence of measures.
First, using the tail balancing condition (7), we have for all $x < 0$,

$$\lim_{n\to+\infty} nP\{a_n^{-1}\zeta_1 < x\} = \lim_{n\to+\infty} (1-\pi_0)nP\{a_n^{-1}|\zeta_1| > -x\} \tag{10}$$

Morever, for all $x > 0$,

$$\lim_{n\to+\infty} nP\{a_n^{-1}\zeta_1 > x\} = \lim_{n\to+\infty} \pi_0 nP\{a_n^{-1}|\zeta_1| > x\}. \tag{11}$$

Using the assumption (8), the expressions (10) and (11), we establish (9) as claimed. ∎

The following lemma permit to get the convergence of the PRM N_n.

Lemma 2 *Assume that the processes (X_t) and (ζ_t) satisfy the hypotheses of lemma 1 and (Y_t) verifies (6) then in the space $\mathcal{M}_p([0,\ \infty)\times(-\infty,\ +\infty)\backslash\{0\})$*

$$N_n \xrightarrow{d} N_1 + N_2, \qquad \textit{as } n \longrightarrow +\infty.$$

Proof : Here we give a modification of the proof of proposition 4.21 of Resnick (1987) applied to linear processes. We must show

$$d\Big(\sum_{k=1}^{\infty} \epsilon_{(\frac{k}{n},\, a_n^{-1}Z_k)},\quad \sum_{k=1}^{\infty} \epsilon_{(\frac{k}{n},\, a_n^{-1}X_k e_1)} + \epsilon_{(\frac{k}{n},\, a_n^{-1}\zeta_k e_2)}\Big) \xrightarrow{P} 0 \tag{12}$$

where $Z_k = (X_k,\ \zeta_k),\quad e_1 = (1,\ 0)$ *and* $e_2 = (0,\ 1)$.

Where d is the vague metric on $\mathcal{M}_p([0,\ \infty)\times(-\infty,\ +\infty)\setminus\{0\})$. It suffices to check for all $f \in C_K^+([0,\ \infty)\times(-\infty,\ +\infty)\setminus\{0\})$ with support contained in $[0,\ 1]\times\{(x_1,\ x_2);\ |x_1| > \delta \textit{ or } |x_2| > \delta\ \}$, for some $\delta > 0$, that

$$I_n(f) - I_n^*(f) \xrightarrow{P} 0, \tag{13}$$

where I_n and I_n^* denote respectively the two terms of (12). Set

$$H \;=\; \{(x_1,\; x_2) \in [-\infty,\; +\infty] : \quad |x_1| > \delta \text{ and } |x_2| > \delta\}.$$

Then,

$$I_n(f) \;=\; \int f\, dI_n \;=\; \int_{[0,\,1]\times H^c} f\, dI_n \;+\; \int_{[0,\,1]\times H} f\, dI_n^*.$$

Since

$$\begin{aligned} E[I_n([0,\;1]\times H)] \;&=\; nP\{a_n^{-1} Z_k \in H\} \\ &\le\; nP\{a_n^{-1}|X_k| > \delta\}\; P\{a_n^{-1}|\zeta_k| > \delta\} \\ &\le\; nP\{a_n^{-1} X_k > \delta\}\; P\{a_n^{-1}|\zeta_1| > \delta\}. \end{aligned}$$

We have:

$$E[I_n([0,\;1]\times H)] \;\longrightarrow\; 0 \quad \text{as } n \longrightarrow\; +\infty,$$

and this readily implies,

$$I_n(f) \;=\; \int_{[0,\,1]\times H^c} f\, dI_n \;+\; o_p(1).$$

Morever, we have obviously

$$I_n^*(f) \;=\; \int_{[0,\,1]\times H^c} f\, dI_n^*.$$

To establish (13), we must show that the following expression

$$\begin{aligned} &\sum_{k=1}^{n} f(\frac{k}{n},\; a_n^{-1} Z_k) 1_{\{a_n^{-1} X_k \le \delta,\; a_n^{-1}|\zeta_k| > \delta\}} \;-\; \sum_{k=1}^{n} f(\frac{k}{n},\; a_n^{-1}\zeta_k e_2) 1_{\{a_n^{-1}\zeta_k > \delta\}} \qquad (14)\\ &+ \sum_{k=1}^{n} f(\frac{k}{n},\; a_n^{-1} Z_k) 1_{\{a_n^{-1} X_k > \delta,\; a_n^{-1}|\zeta_k| \le \delta\}} \;-\; \sum_{k=1}^{n} f(\frac{k}{n},\; a_n^{-1} X_k e_1) 1_{\{a_n^{-1} X_k > \delta\}} \end{aligned}$$

tends to 0 in probability. The first term of (14), which we denote by J_1 can be written in the following form

$$\begin{aligned} \sum_{k=1}^{n} f(\frac{k}{n},\; a_n^{-1} Z_k) 1_{\{a_n^{-1} X_k \le \delta,\; a_n^{-1}|\zeta_k| > \delta\}} \;&-\; \sum_{k=1}^{n} f(\frac{k}{n},\; a_n^{-1}\zeta_k e_2) 1_{\{a_n^{-1} X_k \le \delta,\; a_n^{-1}|\zeta_k| > \delta\}} \\ &-\; \sum_{k=1}^{n} f(\frac{k}{n},\; a_n^{-1}\zeta_k e_2) 1_{\{a_n^{-1} X_k > \delta,\; a_n^{-1}|\zeta_k| > \delta\}}. \end{aligned}$$

We have :

$$|J_1| \;\le\; \sum_{k=1}^{n} |f(\frac{k}{n},\; a_n^{-1}Z_k) \;-\; f(\frac{k}{n},\; a_n^{-1}\zeta_k e_2)| 1_{\{a_n^{-1}X_k \le \delta,\; a_n^{-1}|\zeta_k| > \delta\}}$$

$$+ \sum_{k=1}^{n} f(\frac{k}{n},\; a_n^{-1}\zeta_k e_2) 1_{\{a_n^{-1}X_k > \delta,\; a_n^{-1}|\zeta_k| > \delta\}}. \qquad (15)$$

Let denote by A and B, the two terms of (15).

$$E(B) \;\le\; nP\{a_n^{-1}X_k > \delta\}P\{a_n^{-1}|\zeta_k| > \delta\} \sup f(x).$$

Then

$$E(B) \;\longrightarrow\; 0 \quad \text{as } n \;\longrightarrow\; +\infty.$$

The indicator function associated with A is bounded by $(0 < \eta < \delta)$

$$1_{\{}a_n^{-1}X_k < \eta,\; a_n^{-1}|\zeta_k| > \delta\} \;+\; 1_{\{}a_n^{-1}X_k > \eta,\; a_n^{-1}|\zeta_k| > \eta\}.$$

Therefore

$$E(A) \;\le\; \sup\{|f(s,x) \;-\; f(s,x_2e_2)| : \quad |x_1| \le \eta,\; |x_2| > \delta\} nP\{a_n^{-1}\zeta_k > \delta\}$$

$$+ \;(Constante) nP\{a_n^{-1}X_k > \eta\}P\{a_n^{-1}|\zeta_k| > \eta\}.$$

Since f is uniformly continuous on its compact support, the sup can be made as small as we like by choosing η small. By (8), the bound of $E(A)$ converges as $n \longrightarrow +\infty$ to 0 and hence J_1 tends to 0 in probability. By similar arguments, the second term of (14) which we denote by J_2 tends to 0 in probability and the formulae (12) is proved.

We have proved that

$$\sum_{k=1}^{\infty} \epsilon_{(\frac{k}{n},\, a_n^{-1}Z_k)} \quad \text{and} \quad \sum_{k=1}^{\infty} \epsilon_{(\frac{k}{n},\, a_n^{-1}X_k e_1)} \;+\; \epsilon_{(\frac{k}{n},\, a_n^{-1}\zeta_k e_2)}$$

have the same weak limit behavior.

Since

$$\sum_{k=1}^{\infty} \epsilon_{(\frac{k}{n},\, a_n^{-1}X_k)} \;\xrightarrow{d}\; \sum_{k=1}^{\infty} \epsilon_{(t_k,\, j_k)},$$

and

$$\sum_{k=1}^{\infty} \epsilon_{(\frac{k}{n},\, a_n^{-1}\zeta_k)} \xrightarrow{d} \sum_{k=1}^{\infty} \epsilon_{(t_k,\, l_k)},$$

we have by the continuous mapping theorem

$$\sum_{k=1}^{\infty} \epsilon_{(\frac{k}{n},\, a_n^{-1}X_k e_1)} \xrightarrow{d} \sum_{k=1}^{\infty} \epsilon_{(t_k,\, j_k e_1)},$$

and

$$\sum_{k=1}^{\infty} \epsilon_{(\frac{k}{n},\, a_n^{-1}\zeta_k e_2)} \xrightarrow{d} \sum_{k=1}^{\infty} \epsilon_{(t_k,\, l_k e_2)}.$$

Which implies

$$I_n^* \xrightarrow{d} \sum_{k=1}^{\infty} \epsilon_{(t_k,\, j_k e_1)} \,+\, \epsilon_{(t_k,\, l_k e_2)}.$$

An application of the continuous mapping theorem yield that:

$$\begin{aligned}
\sum_{k=1}^{\infty} \epsilon_{(\frac{k}{n},\, a_n^{-1}Y_k)} \quad &= \quad T_1\Big(\sum_{k=1}^{\infty} \epsilon_{(\frac{k}{n},\, a_n^{-1}Z_k)}\Big) \\
&\approx \quad T_1\Big(\sum_{k=1}^{\infty} \epsilon_{(\frac{k}{n},\, a_n^{-1}X_k e_1)} \,+\, \sum_{k=1}^{\infty} \epsilon_{(\frac{k}{n},\, a_n^{-1}\zeta_k e_2)}\Big) \\
&\xrightarrow{d} \quad T_1\Big(\sum_{k=1}^{\infty} \epsilon_{(t_k,\, j_k e_1)} \,+\, \epsilon_{(t_k,\, l_k e_2)}\Big) \\
&= \quad \sum_{k=1}^{\infty} \epsilon_{(t_k,\, j_k)} \,+\, \epsilon_{(t_k,\, l_k)}.
\end{aligned}$$

To finish the proof, we notice that N_1 and N_2 are independent which is a consequence of the independence of X_t and ζ_t.
Using the Laplace functional, we get

$$\psi_{N_1\,+\,N_2}(g) \;=\; \exp\, -\int_E (1 \;-\; e^{-g(x,\,y)})dm(x,\; y)$$

where $m \;=\; \lambda \times \nu_1$, λ is the Lebesgue measure. This is the desired conclusion. ■

Now we give the main result when F belongs to the Fréchet domain.

Theorem 2 *Under the hypotheses of lemma 2, we have*

$$P\{a_n^{-1}M_n \le x\} \longrightarrow \phi_\alpha(x), \;\; x > 0, \;\; \alpha > 0 \quad as \;\; n \longrightarrow +\infty$$

where $M_n = \max_{1 \le k \le n} Y_k$ *and* (Y_t) *verifies (6).* ■

Proof : Consider the mapping T_2 defined by

$$T_2(\sum_{k=1}^{\infty} \epsilon_{(t_k,\, j_k)}) = \sup\{j_k, \; t_k \le . \}.$$

Set

$$Y_n(t) = \begin{cases} a_n^{-1} M_{[nt]} & \text{if} \quad t \le \frac{1}{n} \quad n \ge 1 \\ a_n^{-1} Y_1 & \text{if} \quad 0 < t < \frac{1}{n} \end{cases}.$$

T_2 is an a.s. continuous mapping from $M_p([0, \infty) \times (0, \infty])$. This relation and the continuous mapping theorem yield that

$$T_2(\sum_{k=1}^{\infty} \epsilon_{(\frac{k}{n},\, a_n^{-1}Y_k)}) =^d Y_n(.) \longrightarrow T_2(\sum_{k=1}^{\infty} \epsilon_{(t_k,\, j_k)} + \epsilon_{(t_k,\, l_k)}).$$

Denote by $Y(.)$ the extremal process limit. By the lemma 1 and 2, we obtain

$$\begin{aligned} P\{Y(t) \le x\} &= P\{N_1 + N_2([0, t) \times [x, \infty)) = 0\} \\ &= \exp\{-\lambda \times \nu_1([0, t) \times [x, \infty))\} \\ &= \exp\{-t x^{-\alpha}\}, \quad x > 0. \end{aligned}$$

■

Theorem 1 permits to show that the asymptotic behavior for the extreme value of (Y_t) defined in (6) belongs also to $D(\phi_\alpha)$. Now if we consider the norming constants

$$\alpha_n = a a_n \quad \text{and} \quad \beta_n = a b_n + b.$$

the result is always true for the process (Y_t) defined in (1) using proposition 1.

2.3 Gumbel domain

Assume now that the distribution function F of X_t belongs to the Gumbel domain, i.e. $F \in D(\Lambda)$ with the norming coefficients a_n and b_n. Here we precise two technical lemma before giving the main result.

Lemma 3 *Let (X_t) be a sequence of iid rvs with common distribution $F \in D(\Lambda)$ and (ζ_t) a sequence of iid rvs with marginal density f_ζ .We assume that (X_t) and (ζ_t) are independent and $a_n \longrightarrow \gamma^{-1} \in (0,\ \infty]$. Then in the space $\mathcal{M}_p([0,\ \infty) \times (-\infty,\ +\infty]^2)$*

$$N_n^{(3)} \ = \ \sum_{k=1}^{\infty} \epsilon_{(\frac{k}{n},\, a_n^{-1}(X_k\, -\, b_n),\, a_n^{-1}\zeta_k)} \ \xrightarrow{d} \ N_3$$

(16)

where N_3 is a PRM($\lambda \times \mu_3$), N_3 can be written in the form $\sum_{k=1}^{\infty} \epsilon_{(t_k,\, U_k,\, V_k)}$, λ is Lebesgue measure on $[0\ ,\infty)$ and $\mu_3(dx, dy) \ = \ e^{-x}dx\gamma^{-1}f_\zeta(\gamma^{-1}y)dy$.

Proof: In view of proposition 3.21 of Resnick(1987), it suffices to check for $(x,\ y) \in (-\infty,\ +\infty]^2$ that

$$nP\{(a_n^{-1}(X_1 - b_n),\ a_n^{-1}\zeta_1) \in [x,\ +\infty] \times [y,\ +\infty]\} \ \longrightarrow \ e^{-x}\bar{F}_{\zeta_1}(\gamma^{-1}y).$$

Since (ζ_t) and (X_t) are independent and $a_n \longrightarrow \gamma^{-1}$, it is now straightforward to obtain the desired result.

Now we establish a convergence result for a point process based on the explained variable (Y_t).

Lemma 4 *Assume the same assumptions than in lemma 3, assume also that (Y_t) verifies (5) and*

$$\theta \ := \ \int_{\mathbb{R}} f_\zeta(t)e^{\gamma t}dt \ < \ +\infty. \tag{17}$$

Then in the space $\mathcal{M}_p([0,\ \infty) \times (-\infty,\ +\infty])$, we get

$$N_n^{(4)} \ = \ \sum_{k=1}^{\infty} \epsilon_{(\frac{k}{n},\ a_n^{-1}(Y_k\ -\ b_n))} \ \xrightarrow{d} \ N_4 \ = \ \sum_{k=1}^{\infty} \epsilon_{(t_k,\ U_k\ +\ V_k)}. \qquad (18)$$

where N_4 is a PRM($\lambda \times \mu_4$), λ is Lebesgue measure on $[0\ ,\infty)$ and $\mu_4(dx) \ = \ \theta e^{-x} dx$.

Proof : An Application of the continuous mapping theorem to (16) gives (18). The computation of the mean measure $\lambda \times \mu_4$ needs (17). As a matter of fact, define the function T from $[0,\ \infty) \times (-\infty,\ +\infty]^2$ into $[0,\ \infty) \times (-\infty,\ +\infty]$ by

$$T(t,\ x,\ y) \ = \ \begin{cases} (t,\ (x+y)) & \text{if} \quad (x,\ y) \in \mathbb{R}^2 \\ (t,\ 0) & \text{if} \quad x = +\infty \text{ or } y = +\infty. \end{cases}$$

By the proposition 2.2 of Davis and Resnick (1988), the mapping

$$\widehat{T} : \ \mathcal{M}_p([0,\ \infty) \times (-\infty,\ +\infty]^2) \ \longrightarrow \ \mathcal{M}_p([0,\ \infty) \times (-\infty,\ +\infty])$$

defined by

$$\widehat{T}(\sum_i \epsilon_{x_i}) \ = \ \sum_i \epsilon_{Tx_i}$$

is continuous. Then

$$N_n^{(4)} \ = \ \widehat{T}(N_n^{(3)}) \ \xrightarrow{d} \ N_4 \ = \ \widehat{T}(N_3)$$

where $\widehat{T}(N_3)$ is a PRM with mean measure $\lambda \times \mu_4$ given by easy computation. Indeed

$$
\begin{aligned}
\mu_4(z,\ +\infty] &= \mu_3 \circ T^{-1}(z,\ +\infty], \\
&= \mu_3\{(x,\ y),\ x+y>z\}, \\
&= \int_{\{x+y>z\}} e^{-x}\gamma^{-1} f_\zeta(\gamma^{-1}y)dxdy, \\
&= \int_{\mathbb{R}} \gamma^{-1} f_\zeta(\gamma^{-1}y) \int_{z-y}^{+\infty} e^{-x}dxdy, \\
&= \int_{\mathbb{R}} \gamma^{-1} f_\zeta(\gamma^{-1}y)e^{-z+y}dy, \\
&= e^{-z}\int_{\mathbb{R}} \gamma^{-1} f_\zeta(\gamma^{-1}y)e^{y}dy.
\end{aligned}
$$

Since the assumption (17) is verified, the result follows upon changing variable.

Theorem 3 *Under the hypotheses of lemma 4, we have*

$$\lim_{n\to+\infty} P[a_n^{-1}(\max_{1\le k\le n} Y_k - b_n) \le x] \ = \ \Lambda^\theta(x),\ \textit{for all}\, x \in \mathbb{R}.$$

where $\Lambda^\theta(x) \ = \ exp(-\theta e^{-x})$. ■

Proof : The proof is identical to that of theorem 2 where $N_1 + N_2$ is replaced by the process limit N_4 defined in lemma 4.

Remark : The parameter θ can be interpreted as the extremal index of the sequence (Y_t). It allows to characterise the relationship between the dependence structure of the data and their extremal behavior. By the same way as in 2.1, we can extend these results to the process (Y_t) defined in (1).

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